

The Brain: Homework for a Theoretical Physics

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Workshop «Theoretical Physics and Mathematics of Brain:
Bridges between disciplines and applications»

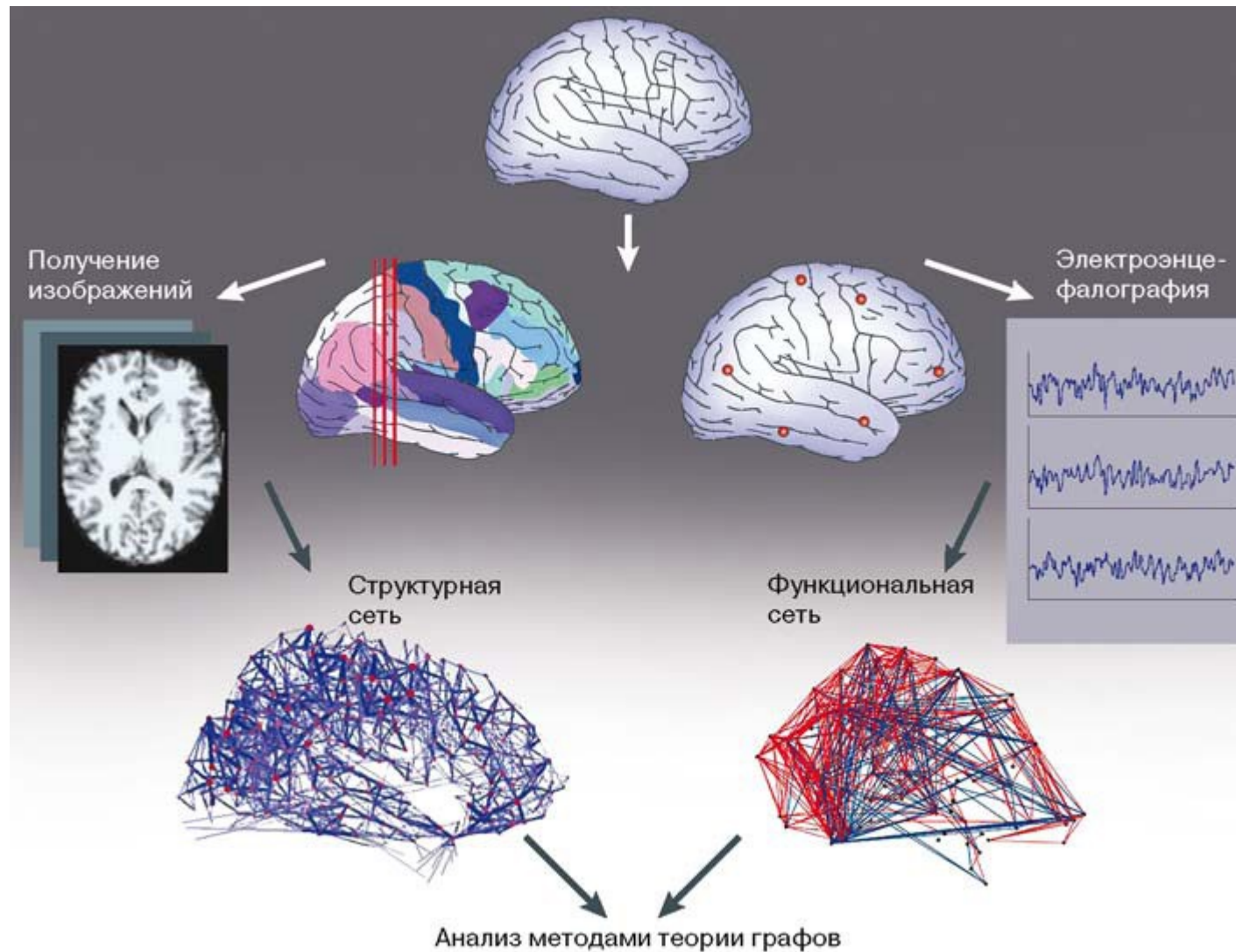
Center for Advanced Brain Studies, MSU , December 4, 2019

Outline of the talk

- Brain as a network. How much knowledge can we get along this line?
- Brain as statistical system. Can we write down the partition function of the brain?
- Is brain a near-critical system? Why and how can we recognize it?
- Can we measure the consciousness ?
- What kind of mathematics do we need to describe a brain?
- My task to cover subject is hopeless of course..

Brain as a network. Connectome
(Sporns +.....)

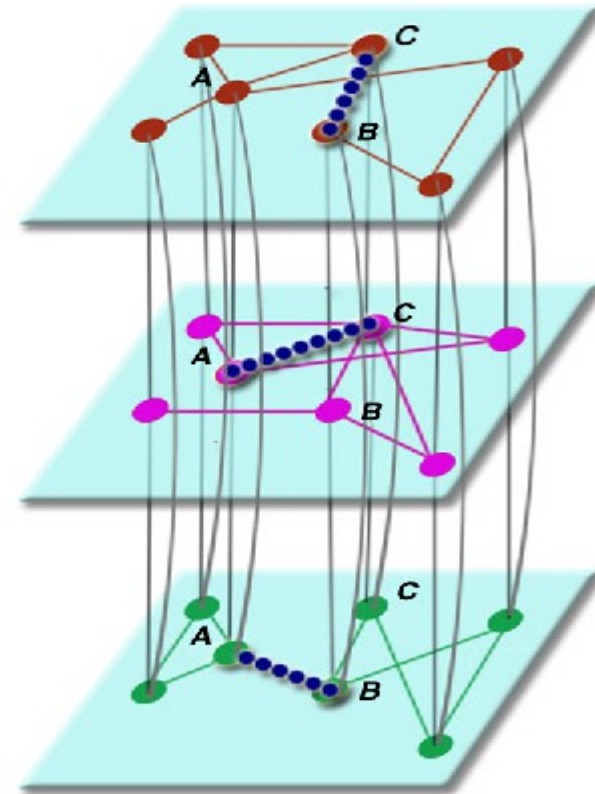
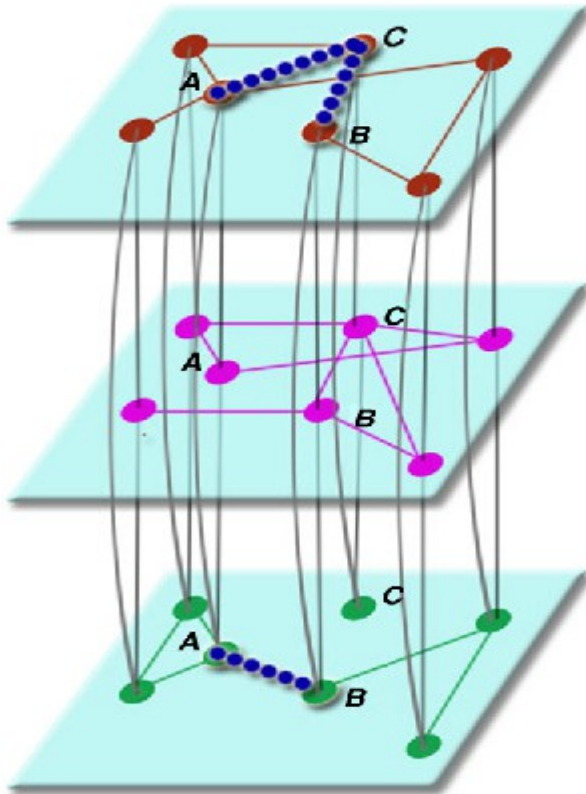
86 bln of nodes, 100 trillions of links in the human brain



Brain as a network

- Structural connectome — anatomy. Nodes—neurons, links — connections between neurons. Completely known only for C-Elegance-302 nodes («Hydrogen atom»). Generically - oriented and weighted network.
- Functional connectome. The set of large number of subgraphs. Each subgraph corresponds to the network of neurons involved in some function(motion,color etc). Each of neurons typically is involved in many subgraphs.

Multiplex networks. Each node is involved into several layers



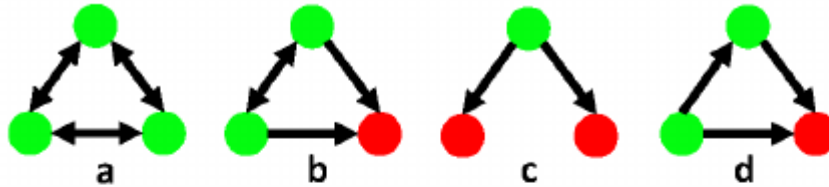
Function I
«color»

Function II
«taste»

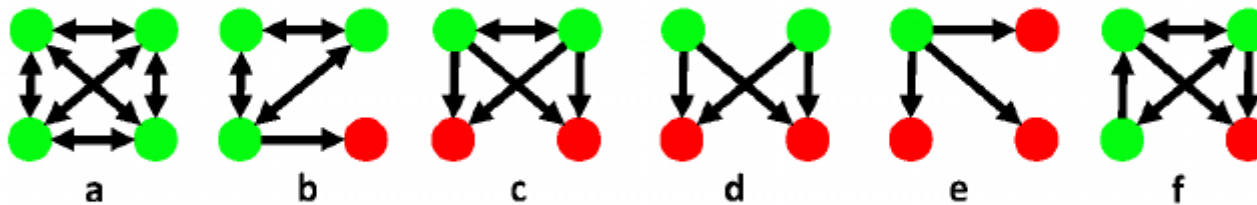
Function III
«place»

Exponential networks — network ensembles with weights for some motifs

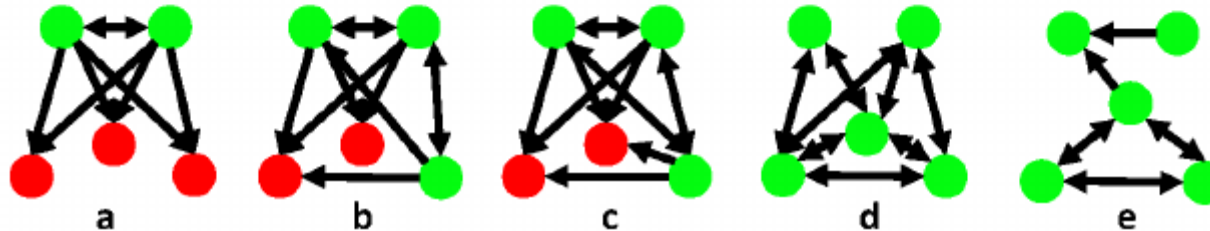
3-node motifs



4-node motifs



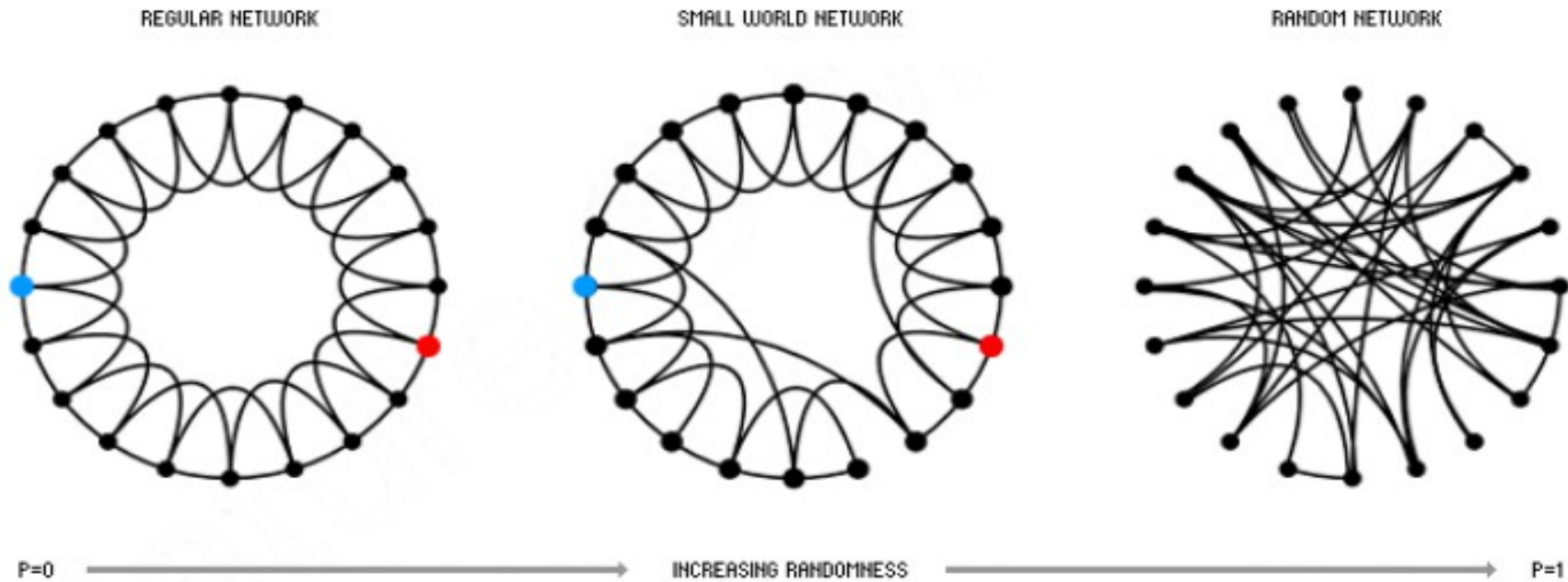
5-node motifs



A lot in common with the matrix models. Special types of matrices. Both single trace and non-single trace terms in the matrix model potential

Connectome. What is known?

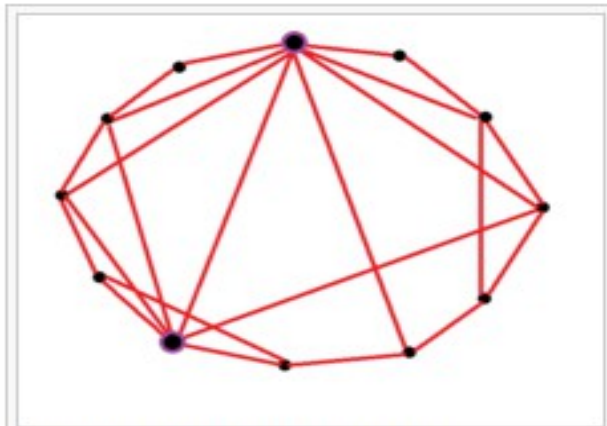
- Small-world network. Hierarchical network
- Average level of clusterization
- Many hubs
- Anomalously large number of «open triads»
- Relations between the diseases and structural changes in connectome
- Complicated synchronization of functional networks



Connectome belongs to very specific type of network architecture, involving elements of small-world network.

Connectome. The Spectral Methods

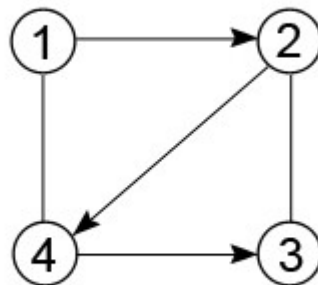
- Connectome — graph, Hence there is the adjacency matrix A and Laplacian of the graph L . The graph Laplacian is a discretization of the 2d Laplacian operator.
- Spectrum of Laplacian of the structural connectome. Spectral density. Spectral correlators. Level spacing distribution.
- The multiplicity of the lowest zero eigenvalue of L - number of disconnected components. The value of the first non-zero eigenvalue — algebraic connectivity.



Пример графа «Мир тесен»,
 выделены вершины — хабы.
 Средняя степень вершины = 1,917
 Средняя длина кратчайшего пути =
 1.803.
 Коэффициент кластеризации = 0.522

Standard network
 characteristics

	1	2	3	4
1	0	1	0	1
2	0	0	1	1
3	0	1	0	0
4	1	0	1	0



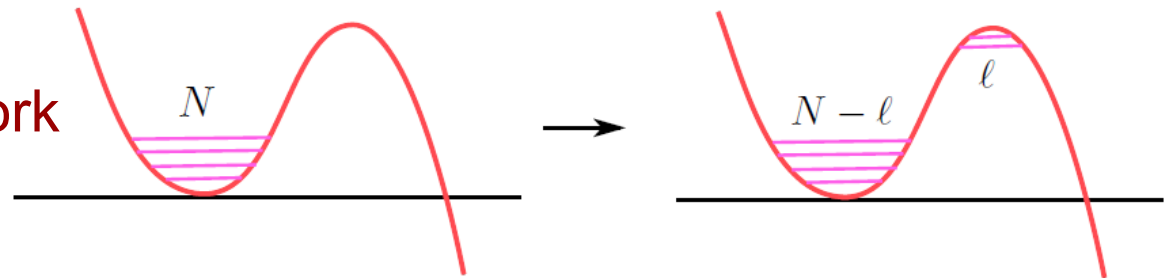
Example of the
 adjacency matrix of the
 graph. A lot of
 information in its spectrum

Spectral methods

- The first nonvanishing eigenvalue of graph Laplacian is proportional to the number of inter-hemispheres links
- The number of isolated low-energy modes is equal to the number of clusters in the network
- The cluster formation is the «nonperturbative effect». Exact analogue of the eigenvalue instanton in the matrix model which corresponds in different situations to formation of domain walls, baby-universes etc.
- More on the spectral analysis in [Pospelov,s talk](#)

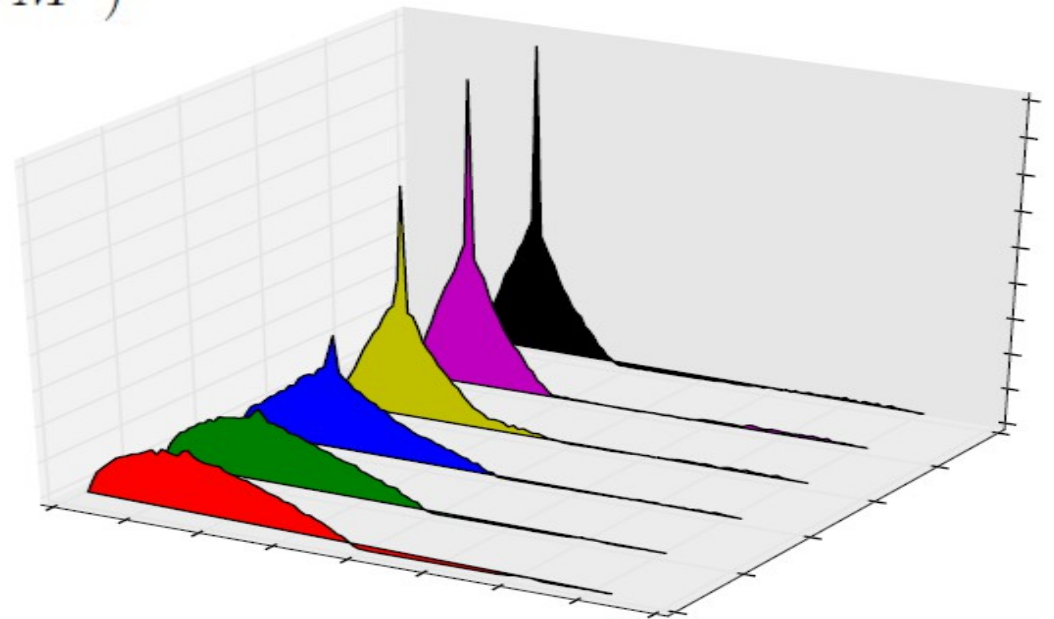
The matrix model counterpart of the cluster formation.
 Eigenvalue tunneling — instanton, nonperturbative
 phenomena in many physical situations.

l clusters emerge in the network



$$Z = \int dM \exp(a \text{Tr} M^2 - \mu \text{Tr} M^3)$$

Example of spectral
 density evolution
 as function of number
 of 3-cycles



Towards the brain partition function
(Friston, Fisher.....)

Model of Friston

- Attempt to write down the free energy as of some statistical system. Based on some general entropic arguments only .
- No Hamiltonian at all. In modern language-topological theory.
- The key idea from the machine learning. Attempt to formulate machine learning as a kind of summation of the entropies of the parts
- To my mind- very artificial approach

Fisher's model (2015)

- They are elementary spin degrees of freedom due to the peculiar objects in the cells — Posner molecule. Idea- spin entanglement of neural qubit-transporter- phosphate ion

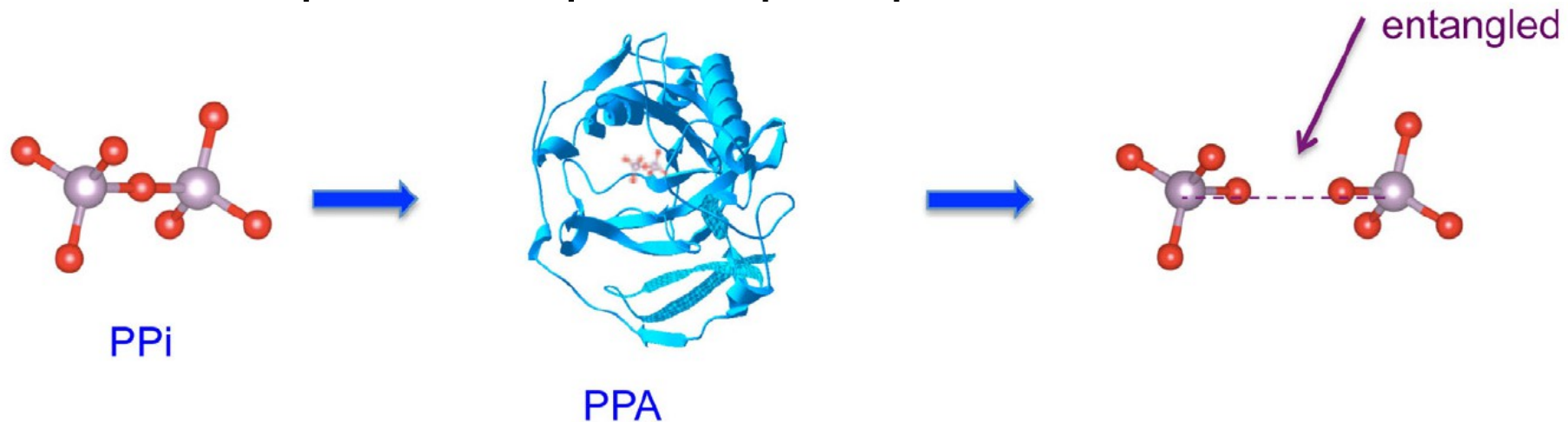


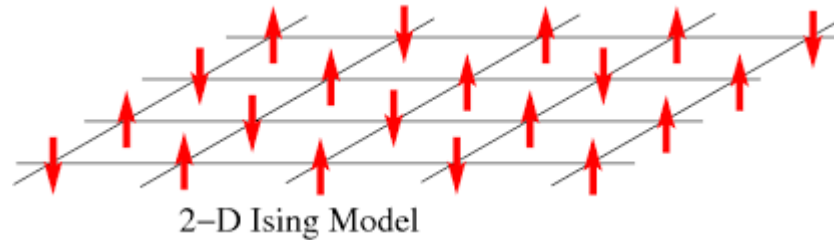
Fig. 3. Pyrophosphate (PPi) is a phosphate dimer with the ^{31}P nuclear spins entangled in a singlet or triplet (analogous to para/ortho hydrogen). As schematized, an enzyme pyrophosphatase (PPA) catalyzes a hydrolysis reaction which splinters pyrophosphate into two separated phosphate ions. The two ^{31}P nuclear spins remain quantum entangled even when separated.

Nuclear spin from the Posner molecules. The only reasonable argument for « quantum physics» involved

The «ground state» at rest

- Ground state of the brain (at sleep) - highly nontrivial with « current of excitations». Any model should respect this aspect.
- The «ground state current» is switched off in the «active state»
- The energy consuming by brain at rest is about 70% compared with the «active state» of brain

Brain «sigma model»?



- The naive starting point. There is some «vector(tensor)» at each node taking value **In higher dimensional stimuli space** . The target space involves the (position in space-time, velocity, color, taste.... etc)
- The **metric of the stimuli space** is quite complicated.
- There is **long-range interaction of the «spins»** at the nodes which yields nontrivial ground state(at rest) with the «current»
- The vector at any node belongs to some representation of the symmetry group acting on the target space. **Representations at all nodes are different and describe the functions this neuron is involved in**
- The **vectors interact with the external field(stimuli)** like Ising spin interacts with the magnetic field

Toy brain «Sigma model»

$$H_{toy} = \sum_{ij} \sum_{ab} A_{ij} C^{ab} S_i^a S_j^b + \sum_{i,a} G_i^a S_i^a$$

A_{ij} - adjacency matrix of the structural connectome

G_i^a - «external field» from the stimuli

C^{ab} - metric in the stimuli space

The «spin» variables S belong to some symmetry group **SU(N) at large N**. At each node the representations of the symmetry algebra are different since different neurons participate the different functional connectomes. **The rank of the symmetry group is time dependent N(t)**. Presumably the simplest version is some generalization of spin glass.

Non-thermalization and conservation laws?

- It seems that there is no ergodization and thermalization of the brain as statistical system
- How the non-thermalization can occur in the brain? Presumably some hidden conservation laws. **Example:** many-body localized phase — **emerging «local conservation laws»** due to interaction. **Each «memory unit »- a kind of emerging conservation law.**
- Surprisingly the conservation of the node degrees and the local connectivity of all nodes play the key role in derivation of the human connectome from the random network.

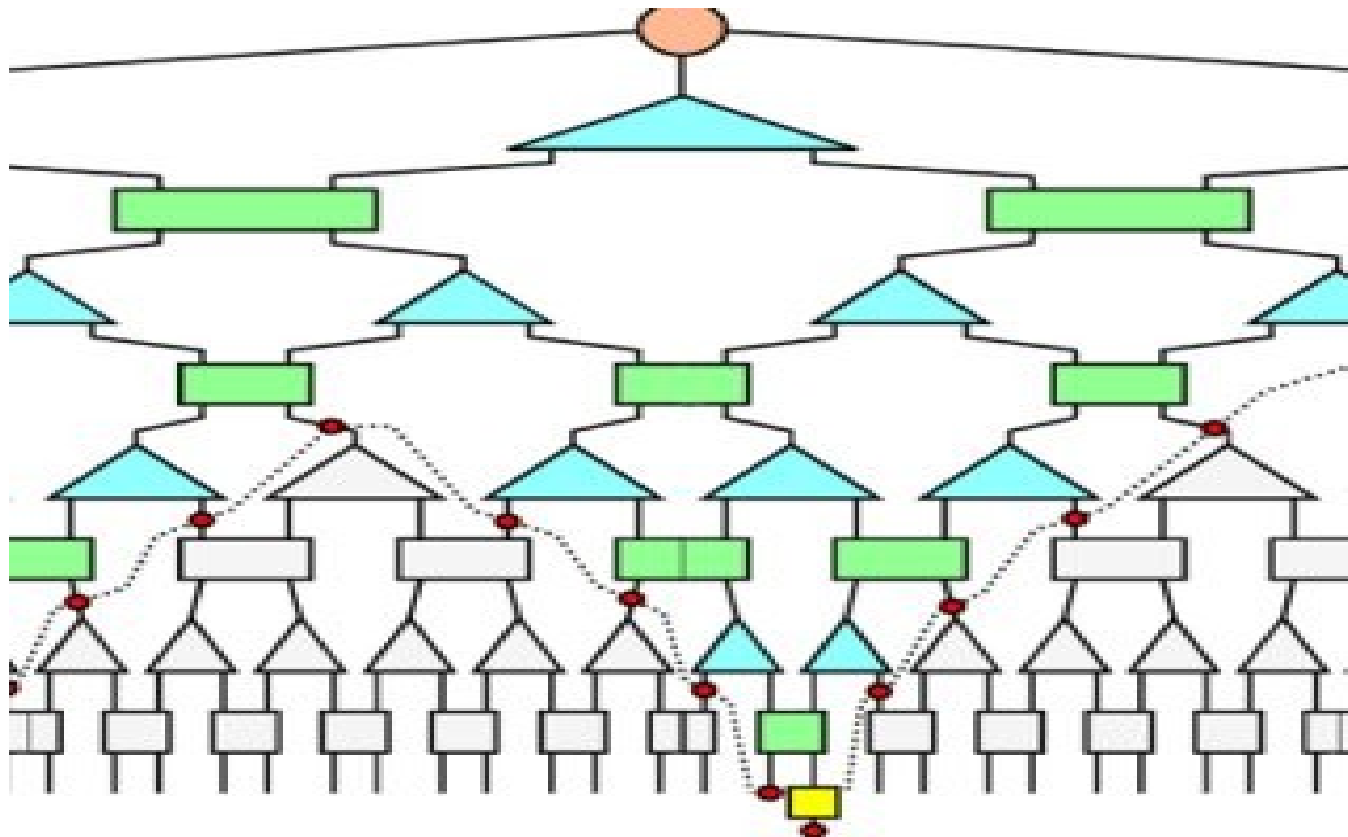
Topological aspects

- There are methods to hunt for the «holes» in the graph based on higher graph homologies. The persistent homologies yield the information about the connectome. Similarly the graph for time series can be analyzed.
- Some functional connectomes(for some stimuli) have the clear-cut non-trivial topology(circle, link). This means that we can discuss topologically nontrivial mapping of the functional connectome into the stimuli space

«Low-energy effective action» and RG flows?

- For generic statistical system usually we look for the low-energy effective action and RG flows
- For brain the most suitable language is the cut-off in the spectrum of Laplacians of the structural and functional connectomes. Low-energy modes- clusters hence the **low-energy dynamics= dynamics of clusters**
- RG flow «from UV to IR» corresponds to (inside the cluster)-> cluster->(union of clusters)

Schematic representation of RG flow



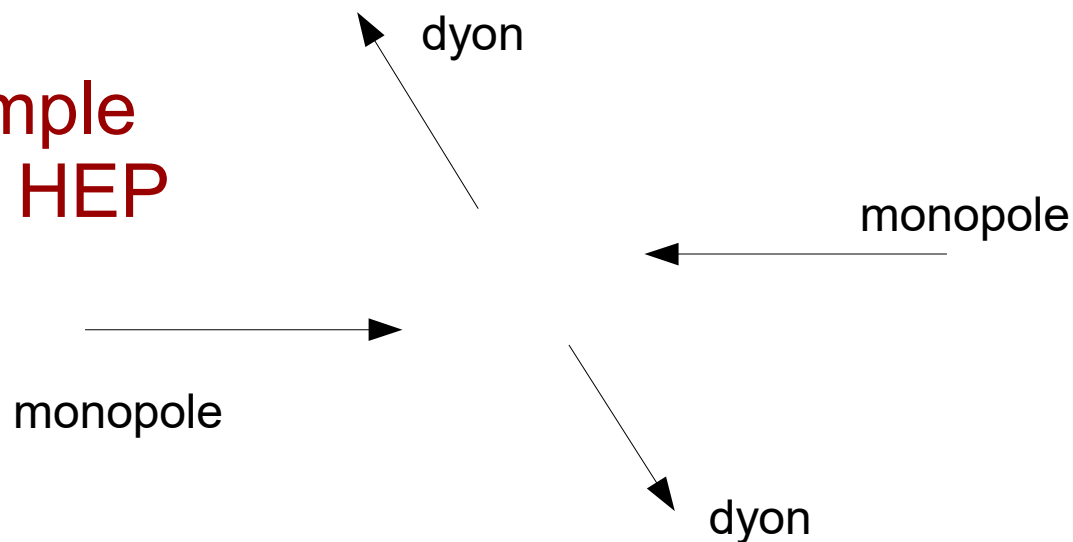
«Low-energy effective action» and RG flows?

- The structural connectome indeed has a kind of hierarchical structure. Not clear if it is true for the functional connectome
- For functional connectome the most suitable language seems to be the scale dependence of the entanglement entropy
- The brain can be considered as the «probe» in the environment. Hence there is the consistency condition as for any probe. In particular the RG flows in the bulk have to be correlated with the RG flows «in the brain».

Geometry of the stimuli space

Target space of the hypothetical «sigma-model» involves unification of the space-time with the different features of the environment like color, sound etc, as well as the «space of human goals». We have to look at the metric of this «moduli space» which seems to be infinite-dimensional. The geodesics in this metric would yield the processes when the space-time gets mixed with the internal stimuli.

Example from HEP



Monopoles get transformed into dyons

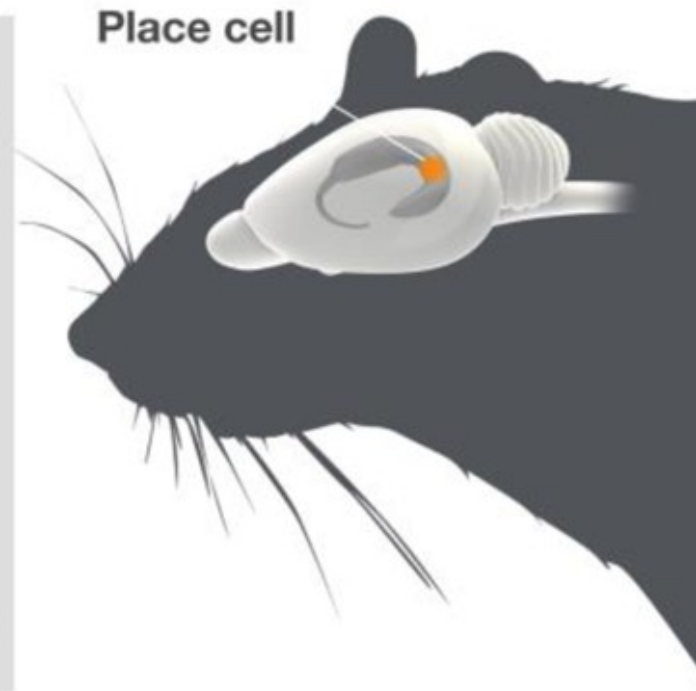
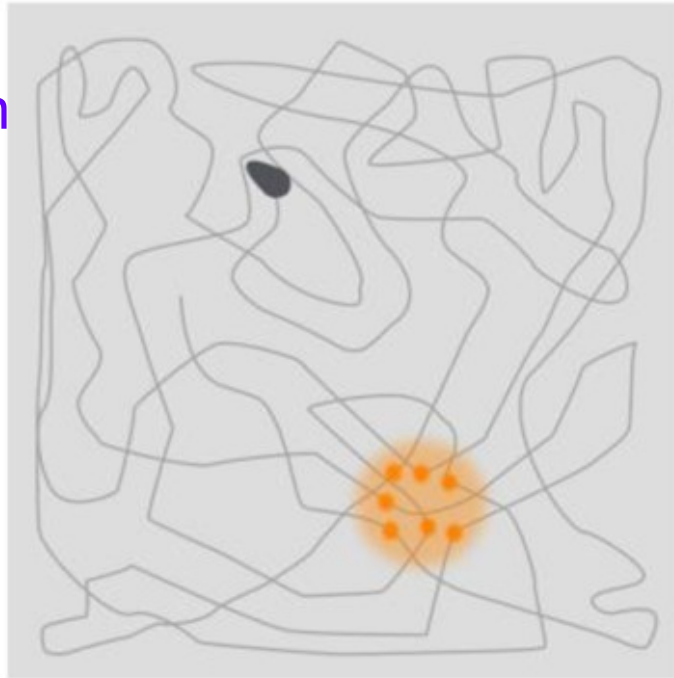
Space gets mixed with charge space

$$g = -dt^2/U(t) + 4l^2U(t)(d\psi + \cos\theta d\phi)^2 + (t^2 + l^2)(d\theta^2 + (\sin\theta)^2 d\phi^2)$$

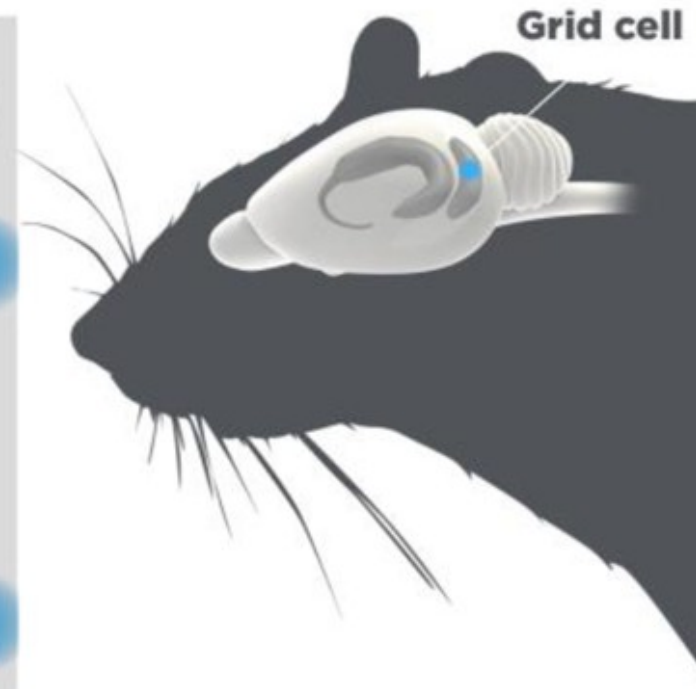
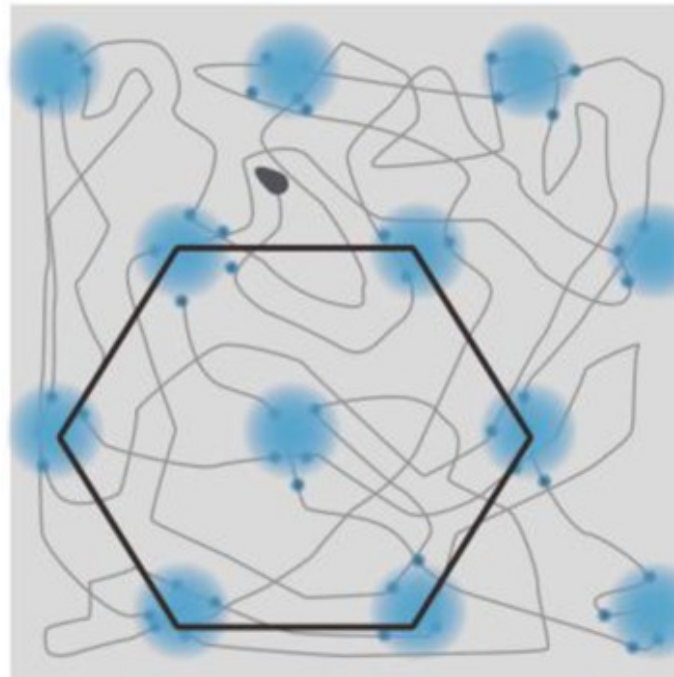
Examples.

- Place cells(GPS) mainly in hyppocampus. Code the position of the animal, direction of motion and its velocity. Provide the memory storage, pattern separation and pattern completion. The theories suggested for GPS are based on the attractor mechanism and spin glass arguments.
- The time cells are also in hyppocampus.It is possible by the external influence to break down the orientation in space or orientation in time separately

Place cells — GPS in brain which are fired at fixed place positions



Grid cells provide the hexagonalization of the external space



Is brain the near -critical system?
(Bialek,.....)

Pro and contra criticality

- It is attractive since it provides the effective response for the external action. There are modes of different wave lengths, no mass scale at criticality
- Evidences; very soft modes in brain rhythms 10-70 Hz, scale-free distributions in avalanches (recently there are questions), critical statistics in the structural connectome

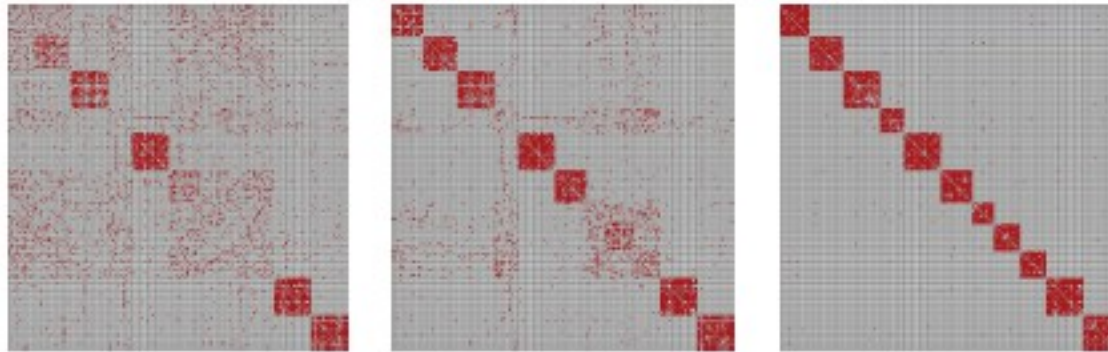
What kind of criticality can be relevant?

- Clusterization phase transition as function of number of cycles
- Synchronization phase transition between layers as the function of strength of interaction
- Anderson-like localization transition for the excitations as the function of network structural disorder
- Measurement induced criticality (Fisher,.....)
- Percolation of motives(open triads and cycles)

Clusterization phase transition in the network driven by 3- cycles

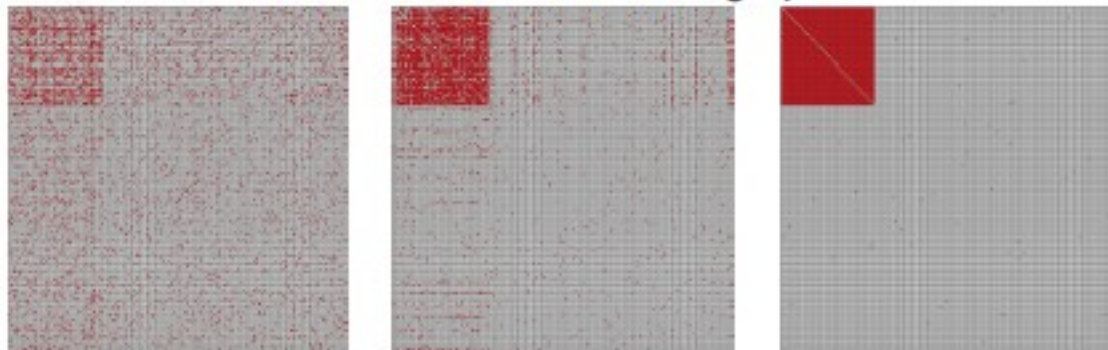
$\text{Tr } A^3$ — number of 3 cycles

kinetics of degree-conserved graphs



(Avetisov, Hovhannisyan
Nechaev, Tamm, Valba,
A.G. '16)

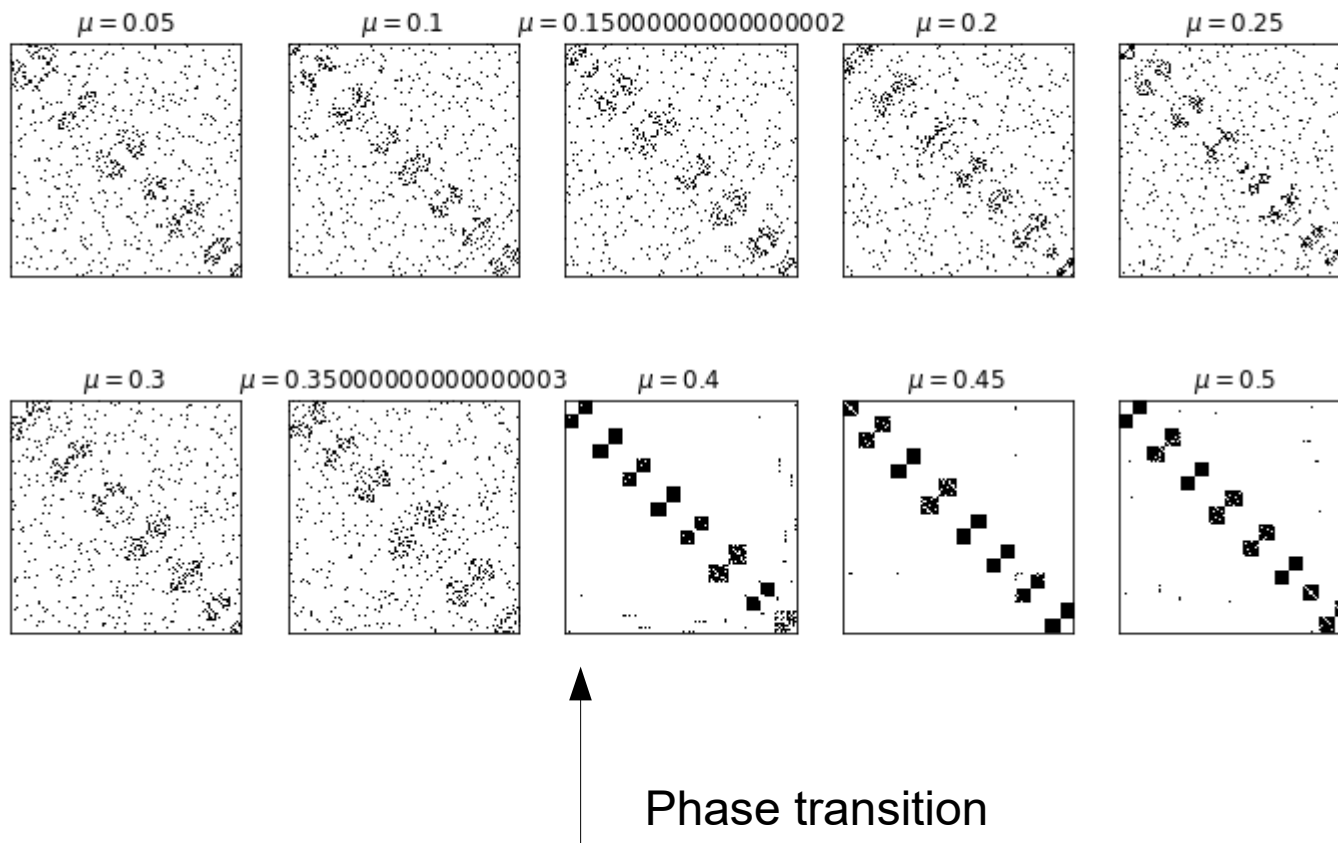
kinetics of non-conserved graphs



Newman, 2003

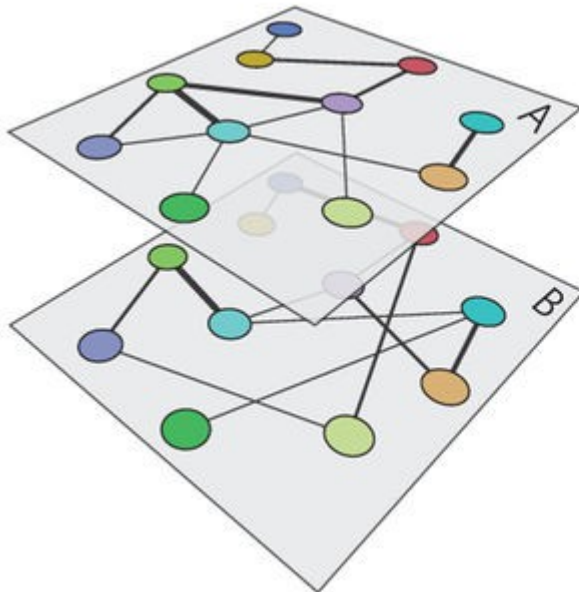
FIG. 5: Few typical samples of intermediate stages of the network evolution: upper panel – evolution with fixed vertex degree; lower panel – evolution with non-fixed vertex degree.

Clusterization phase transition driven by 4-cycles.
The bipartite clusters get formed above the critical point
 $\text{Tr}A^4$ — number of 4-cycles
(O.Valba, A.G. To appear)

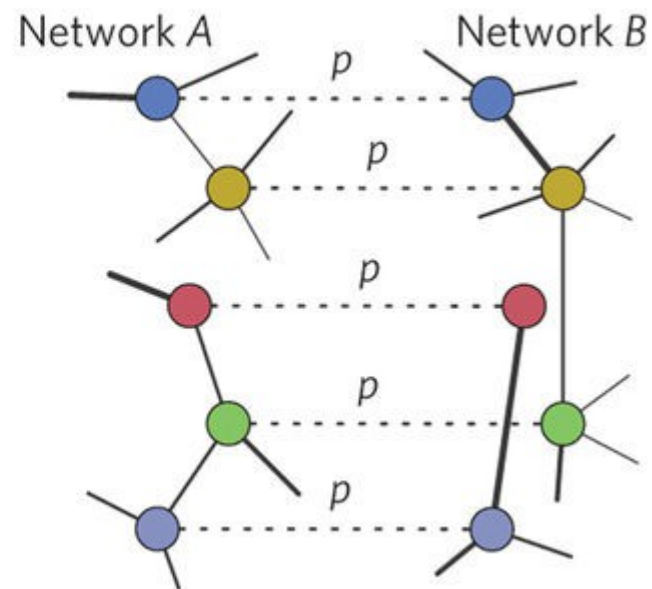


Synchronization of layers

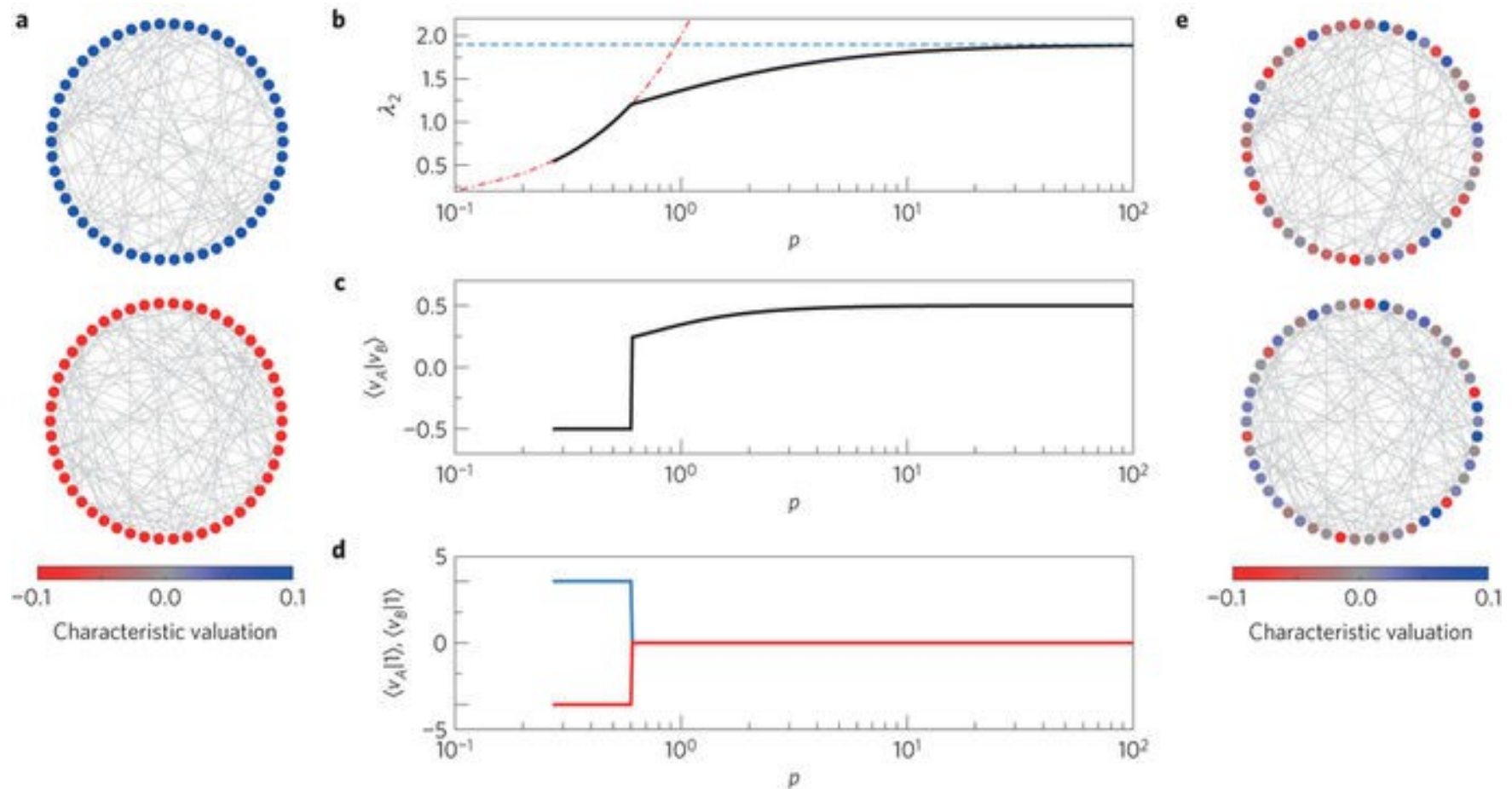
a



b



At some critical value of interlayer coupling p two layers get synchronized. It means that the excitation on the network propagates coherently (Arenas et al).



There are several measures which quantify the synchronization. Mainly the spectral properties.

Criticality in spectral statistics

$$\begin{cases} P_{deloc}(s) = s e^{-as^2} \\ P_{loc}(s) = e^{-s} \end{cases}$$

Level spacing distribution (between the neighbor levels). Two most popular distributions Wigner-Dyson and Poisson corresponds to the delocalization and localization of excitations on the graph.

However there is third type of distributions exactly around the Anderson transition. Very surprisingly the spectral statistics of the non-directed human structural connectome is near-critical. (Pospelov, s talk)

How to measure the consciousness?
(Tononi, Dean.....)

Can we say smth on the consciousness?

- There are two «theories» of consciousness at the market. From my viewpoint both of them are non-satisfactory
- «Theory» of the global neuron space (Dean). Idea behind — self-organized criticality
- «Theory» of integrated information (Tononi). **Two step procedure**. First, find the clusters in the functional connectome. Then evaluate the correlation between clusters. The «order parameter» $\Phi[\text{state}]$ is introduced which has some similarities with the entanglement entropy.

General remarks

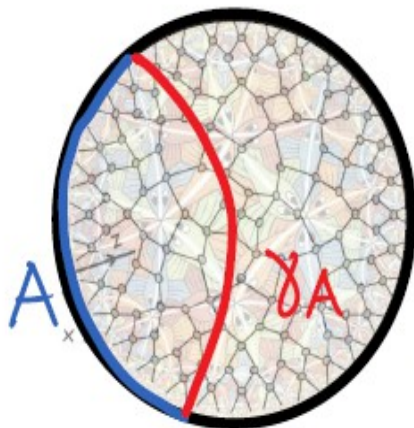
- Many possibilities to attack this issue experimentally. Each day wakeness, anesthesia exit, psychedelics. What happens, phase transition or not?
- **Consciousness is the excited state** of the system, requires more energy. Is there the gap in the spectrum? Does the symmetry of the ground state is different from the symmetry of the conscious state? Very surprisingly the **ground (resting) state involves the nontrivial «current of excitations»** which gets switched off in the conscious state

General remarks

- What is the order parameter? The entanglement entropy could be the candidate. We know examples when the entanglement entropy serves as the order parameter. Two phases: $S \sim \text{Volume}$ or $S \sim \text{Area}$.
- Other possible candidates from the information theory. Entanglement negativity, complexity, Fisher information
- All these characteristics have very clear holographic interpretation. **Example:** Entanglement entropy- the length of geodesic in hyperbolic space.

Radial coordinate in the bulk — RG scale. In the network context radial coordinate — degree of the node.

AdS/CFT duality: conjectural realization

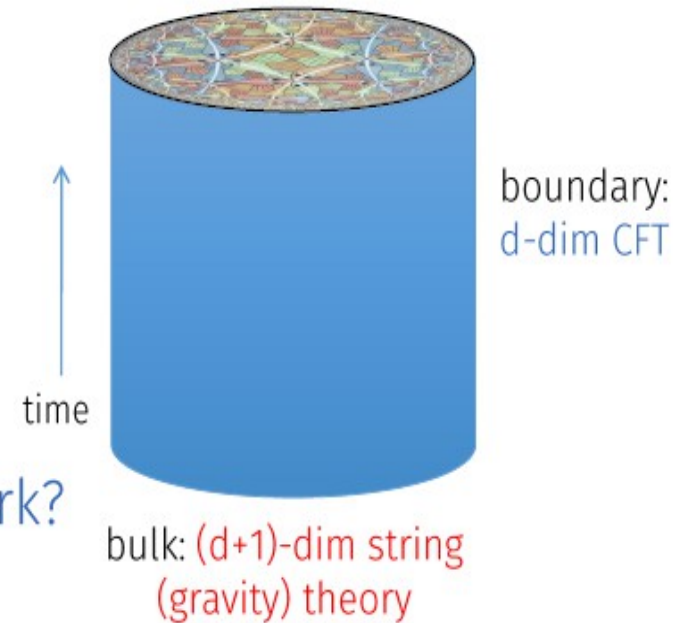


$$S(A) = \frac{1}{4G_N} \min |\gamma_A|$$

[Ryu-Takayanagi]

Space-time as a tensor network?

[Swingle]



General remarks

- The networks are naturally embedded into the hyperbolic space and the very space-time is presumably reconstructed from the networks
- Naive argument. Holographically we consider some probe in $(D+1)$ space-time which capture all information about D space-time. Brain can be considered as a probe if connectome is extended into additional scale coordinate.
- Idea to treat the brain holographically (Dvali). His idea to have bulk geometry with the black hole is highly questionable

One more(naive) remark

- The cat and the human get waked. What is the key difference from the view of «statistical system»? The space of stimuli for the cat is finite and does not change essentially in time. The dimension of the stimuli space for the human is much larger and can be changed
- Hence it seems that the discription of the consciousness should involve the mapping of the network into the target space with varying dimension. Moreover the dimension of the target space is the dynamical variable itself and «Hamiltonian» is time-dependent

What kind of math do we need?

- Graph theory as a whole
- The Category theory
- Knot and link homologies as the source for a entropy
- Persistent graph homologies (topological data analysis) — **more talks at Workshop**
- Some math refinement of the renormalization group like Kontsevich,s attempt via motivic integration - «How to sum up objects?»
- And much more.....

Instead of Conclusion

The situation in neuroscience now reminds the one in the physics of fundamental interactions in 60-th before invention the Standard Model. Amount of experimental data grew very rapidly due to a new generation of accelerators. No reasonable theory. The non-abelian gauge theories did the job. We need «non-abelian gauge theory» for neuroscience.

It is time to attempt

I would like to thank my collaborators V. Avetisov, S. Nechaev, N.Pospelov, O.Valba and especially K. Anokhin for patience shown in introducing me to the subject.