Mathematical and computational models of plasticity and learning in spiking neuronal networks (SNN)

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Nizhny Novgorod city

Moscow
450 km

Siberia,
2000 km
BRAIN INSPIRED INFORMATION PROCESSING SYSTEMS

I. Neuron networks *in silico*: Artificial Neuron Networks (ANNs), Spiking Neuron Networks (SNNs) ("Way of Terminator")

II. Neuroanimat: neuron networks *in vitro*, living cells and tissues as elements of control system ("Way of Robocop")

III. Human-machine neurointerfaces: using of human or animal brain in solving control tasks ("Way of Avatar")
Combining PDMS chips with MEA

Hippocampal cells from Mouse E18

PDMS chip with microchannels

MEA 60 electrodes

“Source” chamber

“Target” chamber

- Dendrites
- Microchannels
- Axons, somas
- Electrodes
- Chambers
Technology “Brain-on-chip”

Microfluidic chip combined with multielectrode array (Sankt Petersburg Academic University)

Pimashkin et al. 2015, “Saint-Petersburg OPEN 2015“
Axons grow from Source (Left) to Target (right) chamber for 3 days. 1 frame was obtained each 20 min. 1 second = 2 hours.
Neurointegration – interaction between pilot nervous system (central and peripheral) and tunable machine controlled to achieve *adaptive result.*
OUTLINE

A. Brain versus digital intelligence. Example of motor control. SNN as a bridge between brain ANN based AI

B. SNN implementing oscillatory associative memory

C. SNN controlling moving robots

D. Unsupervised and supervised learning in SNN: theory and practical application to detect sEMG
Unique brain efficacy in control

Highly coherent activation of large number of muscles on-line

Cluster strategy for motor control (Rodolfo Llinas, 2001)
Grasping movement demands coherent control of more than 50 muscles and muscle synergies. Welsh and Llinas, 1997

Discreteness of the control strategy

To find an optimal one on-line (with 1 msec) a robot demands computations with $10^6$ GHz

$10^{15}$ different combinations

Motion kinematics

100ms
Olivo-cerebellar based neuromorphic system for motor control based on coherent activation of spiking neurons

Features:
- synchronized clusters define motor pattern
- theoretically unlimited number of actuators under control

Olivo-cerebellar network
R.Llinas, 1975.

Analogous chip

Spiking network implementing motor control (V. Kazantsev et al. 2003, 2004)
B. OSCILLATORY MEMORY

Basic principles

Hopfield networks (1982)

• Static binary patterns can be memorized in networks using a Hebbian connection matrix
• Each pattern is associated with a minimum of a gradient function

Oscillatory networks

• Kuramoto’s oscillatory networks with Hebbian connection matrix is used;
• Information is encoded using in-phase and anti-phase locked oscillations between units
• The retrieval state appears as a global mutual phase locking mode corresponding to a memorized pattern configuration
• Information capacity for perfect retrieval $K=2$. 
• **Architecture**

Input layer

Output pattern

Retrieved pattern

Base rhythm

• **Equations**

A spiking oscillator (Rowat & Selverston, 1993)

\[
\begin{align*}
\tau_m \frac{dV_i}{dt} &= -I_{\text{fast}}(V_i) - I_{\text{syn}}(V_i, V_k^0) - W; \\
\tau_w(V_i) \frac{dW_i}{dt} &= W_\infty(V_i) - W_i.
\end{align*}
\]

\(V_i\) and \(V_k^0\) – membrane potentials of post- and presynaptic cells;

\(W_i\) – slow recovery current

Synaptic current:

\[I_{\text{syn}} = g_{\text{syn}} S_\infty(V_k^0)(V_i - V_{\text{syn}})\]

\[S_\infty(V_k^0) = 1/\{1 + 1/\exp[(V_k^0 - \theta_{\text{syn}})/k_{\text{syn}}]\}\]

• **Hebbian connection matrix**

\[
s_{ij} = \frac{1}{N} \sum_{k=1}^{K} \xi_i^k \xi_j^k
\]

\(\xi_i^k = \{+1, -1\}\) – binary pattern encoding
OSCILLATORY MEMORY

Excitatory coupling

\[ V_{\text{syn}} > 0 \]

“Anti-phase” locking

\[ V_i(t) \text{ [a.u.]} \]

\[ \text{time [a.u.]} \]

Y Axis Title

Postsynaptic neuron

Presynaptic neuron
Inhibitory coupling

"In-phase" locking

$V_{syn} > 0$
Spiking phase map

Excitatory synapse

Inhibitory synapse
**Synaptic architecture of the associative memory**

- **Hebbian matrix defines the synaptic reversal potentials of the control layer cells**

  \[
  I_{\text{syn}}(V_i) = \frac{1}{N} \sum_{j=1}^{N} g_{\text{syn}} \left( V_i - V_s \left( \frac{1}{N} \sum_{k=1}^{K} \xi_j \xi_i \right) \right) \frac{1 + \exp \left( - \frac{V^0 - \theta_{\text{syn}}}{k_{\text{syn}}} \right)}{1 + \exp \left( - \frac{V^+ (t) - \theta_{\text{syn}}}{k_{\text{syn}}} \right)}
  \]

  \[
  I_{\text{syn}}(V_i) = \frac{1}{N} \sum_{j=M}^{M} g_{\text{syn}} \left( V_i - V_s \left( \frac{1}{N} \sum_{k=1}^{K} \xi_j \xi_i \right) \right) \frac{1 + \exp \left( - \frac{V^- (t) - \theta_{\text{syn}}}{k_{\text{syn}}} \right)}{1 + \exp \left( - \frac{V^- (t) - \theta_{\text{syn}}}{k_{\text{syn}}} \right)}
  \]

  - \( \xi_i^0 = -1 \)  \( V_{\text{syn}} = +1 \)  \( \text{Anti-phase locking} \)  \( V^-(t) \)
  - \( \xi_i^0 = -1 \)  \( V_{\text{syn}} = -1 \)  \( \text{In-phase locking} \)  \( V^+(t) \)

  - **Phase-synchronized synaptic input**
Memorizing and retrieval

\[ V_{syn} = V_s \left( \frac{1}{N} \sum_{k=1}^{K} \xi_j^k \xi_i^k \right) \]
OSCILLATORY MEMORY

**Input layer units**

\[ V_i^\theta(t) \]

**Retrieving process**

**Output layer units**

\[ V_i(t) \]

**C\[ \phi_i(t) \] time**

A

B

C

Input layer units

Retrieving process

Output layer units

\[ \phi_i(t) \] time
OSCILLATORY MEMORY

Retrieval characteristics

Error-free capacity:

\[ K < 1 + \frac{M}{|M - 2Q_M|} \]

- \( M \) - number of “+1” pixels in the core of the input pattern
- \( Q_M \) – minimum discrepancy between “+1” part of the memorized patterns

Pattern overlap:

\[ m_{kn} = \frac{1}{N} \sum_{j=1}^{N} \xi_k \xi^n \]

Initial overlap

Output overlap

Graph showing the relationship between initial overlap and output overlap for different patterns.

Pattern "0" (white circles), Pattern "1" (black dots), Pattern "2" (black squares).
ANN versus SNN

**BackProp**

- Сигнал ошибки для нейрона \( j \) из слоя \( K \):
  \[ e^K_j = \sum \delta^K \cdot w^K_j \]

- Ошибка (локальный градиент) нейрона \( j \):
  \[ \delta_j = e^K_j \cdot \varphi'(\text{sum}) \]

- Коррекция веса нейрона \( j \):
  \[ \Delta w^j_k = \eta \cdot \delta_j \cdot x^l \]

- Общая ошибка сети:
  \[ E = \frac{1}{2} \sum (d^n - y^n)^2 \]

**STDP**

Markram and Sakmann, 1997; Bi and Poo, 1998; Sjöström et al., 2001
Models

Izhikevich’s model of neuron’s dynamics

\[
\begin{align*}
\frac{dV}{dt} &= 0.04V^2 + 5V + 140 - u + I \\
\frac{du}{dt} &= a(bV - u)
\end{align*}
\]

if \( V \geq +30mV \), \( V = c \)
\( u = u + d \)

Short term synaptic plasticity model of Tsodyks & Markram

\[
\begin{align*}
\frac{dx}{dt} &= \frac{z}{\tau_{rec}} - ux\delta(t - t_{sp}) \\
\frac{dy}{dt} &= -\frac{y}{\tau_l} + ux\delta(t - t_{sp}) \\
\frac{dz}{dt} &= \frac{y}{\tau_l} - \frac{z}{\tau_{rec}} \\
\frac{du}{dt} &= \frac{u}{\tau_{facil}} + U(1 - u)\delta(t - t_{sp})
\end{align*}
\]

Long term synaptic plasticity model: STDP – Spike Timing Dependent Plasticity

\[
\begin{align*}
\frac{dw_{ij}}{dt} &= F_+(w_{ij})x_j(t)\delta(t - t_{sp}^i) - F_-(w_{ij})y_i(t)\delta(t - t_{sp}^j) \\
F_+(w_{ij}) &= \lambda(1 - w_{ij}) \\
F_-(w_{ij}) &= \lambda\alpha w_{ij}
\end{align*}
\]
Robot under control of spiking neural network
Self-learning robots: conditioning

Unconditional stimuli from touching sensors induce robot’s turns to the right or to the left from an obstacle.
Association of signals from ultrasonic and touching sensors

Simultaneous neuron activation leads to potentiation of couplings

Robot can make turn reacting on ultrasonic sensors only, e.g. not touching an obstacle
One of motoneurons generates constant forward movement.

Initially only touching sensors result in robot’s turns. - touching moments

Learning as the result of robot interaction with environment.
Trained robot can avoid obstacles

Robot can adapt itself if ultrasonic sensors exchanged
Robot under control of spiking neural network
Temporal (A) and rate (B) coding scheme for single neuron. $S_{t1-t10}$, $S_{f1-f10}$ – stimulators with temporal and frequency rate parameters correspondently, $N_{1-10}$ – presynaptic neurons, $N_p$ – postsynaptic neuron, $t_j$ – stimulating pulse time for neuron $N_j$. $\Delta t = t_{j+1} - t_j$ – time between pulses.
SNN based EMG interface:
unsupervised learning
SNN based EMG interface: supervised learning
Input and output signals of SNN-classifier. Top Example of two EMG channels recording muscle activity of flexor (red) and panel: extensor (blue). C1-C3 – activity (the membrane potential) of three classifier neurons.
Hebbian learning (in the current work, through triplet-based STDP);

Synaptic competition or competition of inputs (in the current work, through synaptic forgetting)

Neural competition or competition of outputs (in the current work, through lateral inhibition).
WHAT DO NEXT?

• Structural plasticity
• Growing SNN virtual and *in silico*
• Embodiment
• Adaptive network “rewiring” in behavior
THANK YOU!
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